The following is a description of an *research* paper that will be worth about 15% of your final grade. You will be scored out of ten, depending on the quality of what you turn in. **This is a medium length written assignment which will be strictly graded**, based on the following criteria:

- Quality of presentation and writing: clarity, use of appropriate English syntax and grammar;
- Correctness: are the mathematics correct? Are they symbols and notation consistent and appropriate?
- Content and flow: does the paper actually make sense and does it read well? Is it compelling?
- Depth of research: how much work went into investigating the topic and formulating the paper? Does it appear that the author understands the mathematical concepts?

Note: this will be graded strictly. If it is clear that little time was spent, or in-depth thought was not given to the research and exposition, then a zero will be given. Do not expect to score highly if you don’t master the major mathematical terms at play—and many of them are intellectually challenging.

**This activity will be due by the last day of class, Dec. 5th, in class.**

The assignment is based on

**Prompt:** Write a 8–10 page (double-spaced, not including the references) paper which fleshes out any of the topics described below. In particular, I want you to read about the topic (from various legitimate sources), develop a baseline understanding, and then continue refining your understanding of the notation, theorems, and computations involved. I want you to demonstrate to me that you deeply understand the use/application of Math 295 concepts in the topic of your choosing. Beyond this, you may be creative with your exposition. This paper must find a balance between exposition and mathematics, i.e., it cannot *just* be mathematics, nor can it *just* be explanatory/historical.

These prompts are meant to be vague, general “springboards”. You have a lot of flexibility within each prompt, and many of these prompts are related. You can venture out from the direct topic you choose below, as long as the principal theme involves your choice of prompt. I would highly recommend coming to talk to me about the paper after you’ve done some preliminary research but before you begin writing it.
• *Fundamentals of Set Theory*—Zermelo-Fraenkel set theory is the modern standard for mathematics. This is the axiomatic framework upon which set theory, and the rest of modern mathematics rests. In this prompt, you’ll explore the history of axiomatic systems. What necessitated axiomatization? What exactly is ZF set theory? Are there are other axiomatic frameworks? What are benefits and pitfalls of this axiomatization, or any axiomatization.

[Active Link 1] [Active Link 2]

• *Constructing the Integers*—In the first week we talked a lot about Bertrand Russell. Russell was interested in set theory, and in particular, he wrote about Peano’s construction of the integers from a consistent set of mathematical principles known as set theory. In this prompt you’ll investigate this story—what lead people like Peano and Russell to try to construct the integers rigorously? How did these constructions work? Did they ultimately succeed? There is a lot of freedom here, but you should focus on those mathematicians and logicians mentioned here and their immediate predecessor and successors. This topic has a lot of overlap with many of the others here.

[Active Link 1] [Active Link 2]

• *The Axiom of Choice*—The Axiom of Choice is typically appended to ZF Set theory (see above) to yield the major foundations for modern mathematics. It is a controversial axiom in some contexts. On the one hand, without it, much of mathematics does not proceed. On the other, if we take it as an axiom, it often leads to “odd” places. Your job in this prompt is to flesh out exactly what it is. Where did it come from? What do we “need” it for, and what weird consequences result from it?

[Active Link 1] [Active Link 2]

• *Incompleteness*—In the first week of class we discussed Hilbert’s desire to have a consistent and complete set of axioms that would lead to the construction of the integers. In a reasonable world, we’d expect this to be possible. However, the startling revelation made by Kurt Godel flew in the face of mathematicians like Bertrand Russell who believed Hilbert’s problem to be solve-able. In Godel’s two incompleteness theorems he addresses this, and shows, in fact, that any consistent system robust enough to support the integers cannot be complete. Your task in this prompt is to investigate these theorems, their context, and try to understand why Godel’s proof was such a “dagger” to how mathematicians viewed the world. This is not an easy prompt, but you can talk informally about some of the ideas and concepts, and you are allowed to discuss the historical circumstances leading to Godel’s discovery and the ramifications of it. This is also related to the prompt below, and many other prompts.

[Active Link 1] [Active Link 2] [Active Link 3]
• *The Continuum Hypothesis*—In our discussions we have talked about cardinalities, functions, and infinite sets. We already know that there is not just one “notion” of infinity. The continuum hypothesis is a conjecture that there is no cardinality (cardinal number) between $|\mathbb{Z}|$ and $|[0,1]|$. As it turns out, this problem is hard...and its resolution involves the notion of incompleteness in the above prompt. Your job in this prompt is to explain the continuum hypothesis in context and the resolution about whether it is true or not. If you choose this prompt, your paper needs to be detailed and technically precise. BUT you will have the opportunity to use many of the concepts about sets and functions from class.

[Active Link 1]  [Active Link 2]

As with any research oriented paper, in text citations should be utilized, and a detailed bibliography present. (The style of annotation is not important to me, as long as it is consistent and one of the standard citation formats.) Forums and Wiki-based websites *do not constitute suitable sources for citation*, although you may certainly use them to get started. There are resources available through the library for assistance with research and citations: [Active Link]