Activity 1
MATH 120, Fall 2016

Name (Please print):_______________________________

This is a graded activity. You may work in groups, but please turn in your own paper. The first page of material is reference material, **please read that first!**

Reference Material

- We say that
  \[ \lim_{x \to a} f(x) = L \]
  when (informally) the values of \( f(x) \) can be made as close to \( L \) as we would like by considering those \( x \) values which are sufficiently close to (on either side) \( x = a \). Even more informally: as the \( x \) values get close to \( a \), the \( y = f(x) \) values get close to \( L \).

- Recall: we do not actually care about what happens at \( f(a) \), we are trying to describe what happens to \( f(x) \) as \( x \) is “near” \( a \).

- As we discussed in class, we can also talk about left and right hand limits, which only consider \( x \) approaching \( a \) from the left side \( \lim_{x \to a^-} \) or right side \( \lim_{x \to a^+} \).

- More quantitatively, the formal definition of a limit is as follows:
  \[ \lim_{x \to a} f(x) = L \]
  means:
  
  For any (small) number \( \epsilon > 0 \), we can find a (small) number \( \delta > 0 \) so that:
  
  if \( |x - a| < \delta \) then we know (for certain) that \( |f(x) - L| < \epsilon \).

  Recall, in this context \( |c - d| \) measures the *distance* between \( c \) and \( d \).

- The following are the so-called *limit laws* that can help in the evaluation of limits:
  
  Let \( \lim_{x \to a} f(x) = L \) and \( \lim_{x \to a} g(x) = M \), and let \( c \) be a constant:
  
  \[ \lim_{x \to a} [f(x) \pm g(x)] = L \pm M \]
  \[ \lim_{x \to a} [cf(x)] = cL \]
  \[ \lim_{x \to a} [f(x)g(x)] = L \cdot M \]
  \[ \lim_{x \to a} \frac{f(x)}{g(x)} = \frac{L}{M} \text{, so long as } M \neq 0. \]
Compute the following limits, and provide justification for your work in a neat and orderly fashion.

1. \[ \lim_{{x \to 2}} \frac{x^2 + x - 6}{x - 2} \]

2. \[ \lim_{{x \to 3}} \frac{x - 2}{x^2 + x - 6} \]

3. \[ \lim_{{x \to 3}} \frac{x^2 + x - 6}{x - 2} \]

4. \[ \lim_{{x \to 2}} \left[ \frac{\sin \left( \frac{7\pi x}{6} \right) + e^x}{x^2 + \pi} \times \frac{x^2 + x - 6}{x - 2} \right] \]
5. Let \( f(x) = \frac{|x|}{x} \).

(a) \( \lim_{x\to 1} f(x) \)

(b) \( \lim_{x\to -1} f(x) \)

(c) \( \lim_{x\to 0} f(x) \)

6. \( \lim_{x\to -1} \frac{1}{(x+1)^2} \)

7. Let \( f(x) = \frac{\sin(x)}{x} \).

Estimate \( \lim_{x\to 0} f(x) \) by considering the following sequence of \( x \)-values: \( \{\pi/2, .5, .25, .1, .01, .001\} \) (use a calculator to evaluate the function at these points).
8. \( \lim_{t \to 0} \left[ \frac{1}{t} - \frac{1}{t^2 + t} \right] \)

9. \( \lim_{x \to 1} \frac{1 - \sqrt{x}}{1 - x} \)

10. \( \lim_{\theta \to 0} \frac{\sin(2\theta)}{\sin(\theta)} \)