Section 3.1

Problem 1
(a) (i) \((u \in A) \Rightarrow (u \in B)\).
   (ii) \((x \in \mathbb{R}) \Rightarrow (x^2 \geq 0)\).
   (iii) \((t \in \mathbb{N}) \Rightarrow (w \mid t)\).
(b) (i) If \(u \in A\), then \(u \in B\).
   (ii) If \(x \in \mathbb{R}\), then \(x^2 \geq 0\).
   (iii) If \(t \in \mathbb{N}\), then \(w \mid t\).
(c) (i) \(A\) is a subset of \(B\).
   (ii) The square of every real number is nonnegative.
   (iii) \(w\) divides every natural number.

Problem 3
(a) \((\forall x, y \in S)(x + y \in S)\).
(b) \((\forall S \in U)[(|S| = n) \Rightarrow (|\mathcal{P}(S)| = 2^n)]\).
(c) \((\forall x \in \mathbb{Z})[3 \mid (x + (x + 1) + (x + 2))]\).

Problem 4
(a) \(\exists x \in A\).
(b) \(\exists a \in \mathbb{Z})(x = 2^n)\).
(c) \(\exists x_0 \in \mathbb{R})(x_0^2 = 2)\).
Problem 5
(a) (i) \(x\) is the free variable.
   (ii) \(A, B,\) and \(C\) are the free variables.
   (iii) \(c\) is the free variable.

(b) (i) \(x = 1\) makes the sentence true.
    (ii) \(A = \{1\}, B = \{2\},\) and \(C = \{1, 2\}\) make the sentence true.
    (iii) \(c = t^2\) makes the sentence true.

(c) (i) \(x = \frac{1}{2}\) makes the sentence false.
    (ii) \(A = \{1\}, B = \{2\},\) and \(C = \{3\}\) make the sentence false.
    (iii) \(c = 1\) makes the sentence false.

Problem 6
(a) \((\exists a \in \mathbb{N}) [b \mid a \text{ and } c \mid a \text{ and } (bc) \nmid a]\).

(b) \((\exists x, y \in \mathbb{N}) [a \mid (xy) \text{ and } a \nmid x \text{ and } a \nmid y]\).

(c) \((\forall x, y \in \mathbb{Z}) [ax + by \neq c]\).

(d) \((\exists u \in \mathbb{R}) [u \leq x \text{ and } u > x^2]\).

(e) \((\forall t \in \mathbb{N}) (c \nmid t^2 \text{ or } c \mid t)\).

Problem 8
(a) \(\mathcal{T} = \{(b, c) \in \mathbb{N}^2 \mid \text{GCD}(b, c) = 1\}\).

(b) \(\mathcal{T} = \{a \in \mathbb{N} \mid a \text{ is prime or } a = 1\}\).

(c) \(\mathcal{T} = \{(a, b, c) \in \mathbb{Z}^3 \mid c \text{ is a multiple of } \text{GCD}(a, b)\}\).

(d) \(\mathcal{T} = (\infty, 0] \cup [1, \infty)\).

(e) \(\mathcal{T} = \{c \in \mathbb{N} \mid c = a^2b \text{ for some } a, b \in \mathbb{N}\}\).

Problem 9
(a) If a set has \(n\) elements, then its power set has \(2^n\) elements.

(b) The function \(f\) is not strictly increasing on \(\mathbb{R}\).

(c) \(t\) is not in the set \(\text{Im}(f)\).

(d) \(a\) does not divide \(b\).