1. Consider the function \( f(x) = x^4 - 5x^2 + 4. \)

(a) Find the local extrema using the second derivative test. Show ALL steps and justify your conclusions in order to get full credit.

First find the critical values (where extrema might occur).

\[
\frac{d}{dx} f(x) = 4x^3 - 10x = x(4x^2 - 10)
\]

Set equal to zero:

\[
0 = \frac{d}{dx} f(x) = x(4x^2 - 10)
\]

\( x = 0 \) or \( 4x^2 - 10 = 0 \) \( \iff \) \( x^2 = \frac{10}{4} \)

\( \iff \) \( x = \pm \sqrt{\frac{5}{2}} \)

Now check concavity at all critical values:

\[
\frac{d^2}{dx^2} f(x) = 12x^2 - 10; \quad \frac{d^2}{dx^2} f(0) = -10 \Rightarrow \text{concave down}
\]

\[
\frac{d^2}{dx^2} f\left(\sqrt{\frac{5}{2}}\right) = 12 \cdot \frac{5}{2} - 10 = 20 \Rightarrow \text{concave up at both critical values}
\]

\[
\frac{d^2}{dx^2} f\left(-\sqrt{\frac{5}{2}}\right) = 12 \cdot \left(-\sqrt{\frac{5}{2}}\right) - 10 = 20
\]

Hence, \( \left(\sqrt{\frac{5}{2}}, f\left(\sqrt{\frac{5}{2}}\right)\right) \) and \( \left(-\sqrt{\frac{5}{2}}, f\left(-\sqrt{\frac{5}{2}}\right)\right) \) are local minima, \( (0, f(0)) \) is a local max.
(b) Find the points of inflection for \( f(x) = x^4 - 5x^2 + 4 \). Show ALL steps and justify your conclusions in order to get full credit. (You may reference calculations you completed on the front without repeating them.)

To find inflection pts, see where the concavity (second deriv.) changes sign.

From before

\[ f''(x) = 12x^2 - 10 \]

set \( f''(x) = 0 \)

\[ 0 = 12x^2 - 10 \] (solve)

\[ x^2 = \frac{10}{12} \Rightarrow x = \pm \frac{\sqrt{5}}{3} \]

Plot sign of \( f'' \)

\[ f''(-1) = + \]
\[ f''(0) = - \]
\[ f''(1) = + \]

Thus both \( (\sqrt{\frac{5}{12}}, f(\sqrt{\frac{5}{12}})), (-\sqrt{\frac{5}{12}}, f(-\sqrt{\frac{5}{12}})) \) are inflection points.