11/5/2015

Quiz 5

MATH 105, Fall 2015

Name (Please print): KEY

Answer the following questions, showing all work where appropriate **on other sheets.** You can work with others, but you must turn in your own work.

1. Consider the function $f(x) = x^4 - 5x^2 + 4$.

   (a) Find the local extrema using the second derivative test. Show ALL steps and justify your conclusions in order to get full credit.

First find the critical values (where extrema <u>might</u> occur).

$$f'(x) = 4x^3 - 10x = x(4x^2 - 10)$$

Set equal to zero:

$$0 \overset{set}{=} f'(x) = x(4x^2 - 10)$$

$$x = 0 \quad \text{or} \quad (4x^2 - 10) = 0 \iff x^2 = \frac{10}{4}$$

$$\iff x = \pm\sqrt{5/2}$$

Now check <u>concavity</u> at all critical values:

$$f''(x) = 12x^2 - 10; \quad f''(0) = -10 \Rightarrow \text{concave down}$$

$$f''(\sqrt{5/2}) = 12 \cdot \frac{5}{2} - 10 = 20$$

$$f''(-\sqrt{5/2}) = 12 \cdot 5/2 - 10 = 20$$

$\Rightarrow$ concave up at both critical values

Hence, $(\sqrt{5/2}, f(\sqrt{5/2}))$ and $(-\sqrt{5/2}, f(-\sqrt{5/2}))$ are local minima, $(0, f(0))$ is a local max.

(b) **Find the points of inflection for $f(x) = x^4 - 5x^2 + 4$. Show ALL steps and justify your conclusions in order to get full credit. (You may reference calculations you completed on the front without repeating them.)**

To find inflection pts, see where the concavity (second deriv.) <u>changes sign</u>.

From before

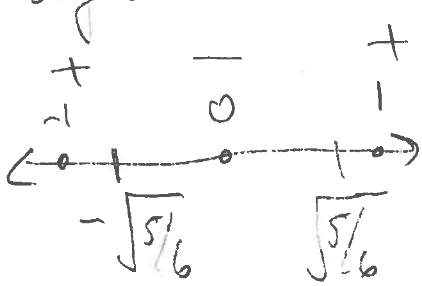
$$f''(x) = 12x^2 - 10$$

set $f''(x) = 0$

$$0 = 12x^2 - 10 \quad \text{(solve)}$$

$$x^2 = \frac{10}{12} \Rightarrow x^2 = \frac{5}{6} \Rightarrow x = \pm\sqrt{5/6}$$

Plot sign of f''



$f''(-1) = +$

$f''(0) = -$

$f''(1) = +$

Thus both $\left(\sqrt{5/6}, f(\sqrt{5/6})\right), \left(-\sqrt{5/6}, f(-\sqrt{5/6})\right)$ are inflection points.