NAME: KEY

Please read the questions. Show all of your reasoning and calculations (the more you show, the more I can give you points for). Try to be organized and clear. Solutions/answers without work are not worth many points. You do not need to simplify your answers unless explicitly told to do so. This is a closed note/closed book test. Calculators capable of symbolic manipulation are not allowed. GOOD LUCK!

Problem	**Points**	**Score**
1	14	
2	7	
3	5	
4	6	
5	5	
6	5	
7	4	
8	8	
9	4	
10	5	
11	7	
Bonus	+5	
Total	70	

1. (14 pts) Compute $f'(x)$ for the following functions. Show all work, and when in doubt, show each step in your calculation.

(a) $f(x) = e^2 + x^2 + e^{-x}$

$$f'(x) = (e^2)' + (x^2)' + (e^{-x})'$$
$$= 0 + 2x + e^{-x}(-1) = 2x - e^{-x}$$

(b) $f(x) = e^{x^2-x+1} = \exp(x^2 - x + 1)$

$$f'(x) = \exp(x^2-x+1) \cdot [x^2-x+1]'$$
$$= (2x-1)e^{x^2-x+1}$$

(c) $f(x) = \ln(2 + x)$

$$f'(x) = \frac{1}{2+x}[2+x]' = \frac{1}{2+x}$$

(d) $f(x) = \ln(3 \cdot x)$

$$f'(x) = \frac{1}{3x}[3x]' = \frac{1}{x}$$

(e) $f(x) = \ln\left(x^{1/10}\right)$ $= \frac{1}{10}\ln(x)$

$$f'(x) = \frac{1}{10} \cdot \frac{1}{x}$$

(f) $f(x) = (\ln(x))^{1/2}$

$$f'(x) = \frac{1}{2}(\ln(x))^{-1/2} \cdot \frac{1}{x}$$

(g) $f(x) = \frac{\ln(x)}{x}$

$$f'(x) = \frac{x(1/x) - \ln x(1)}{x^2} = \frac{1 - \ln(x)}{x^2}$$

2. (7 pts) Find an equation of the line tangent to $y = \ln(x) \cdot e^x$ at $x = 1$.

$$\frac{dy}{dx} = \ln(x)e^x + \frac{1}{x}e^x$$

$$m_{tan} = \left.\frac{dy}{dx}\right|_{x=1} = \ln(1)e^1 + \frac{1}{1}e^1 = 0 + e = e$$

$f(1) = \ln(1)e^1 = 0$ so $\boxed{y - 0 = e(x-1)}$

3. (5 pts) Consider a function $f(x)$ such that $f''(x) = \dfrac{(x-1)^2(x+2)}{x^2+1}$.

Identify the x-value(s) of the inflection points of f and make sure to provide a justinification for your answer.

sign of f''

$-$ $+$ $+$

-2 1

$f''(-3) = -$

$f''(0) = +$

$f''(2) =$

HCVs: $x = 1$, $x = -2$

Since f'' changes sign at $x = -2$, f changes concavity there. Thus $(-2, f(-2))$ is an infl. pt.

4. (6 pts) Compute the hypercritical value(s) for the function $f(x) = (x+1)e^x$.
(Recall that $e^x > 0$ for all x values.)

$$f(x) = xe^x + e^x$$

$$f'(x) = e^x + xe^x + e^x = 2e^x + xe^x$$

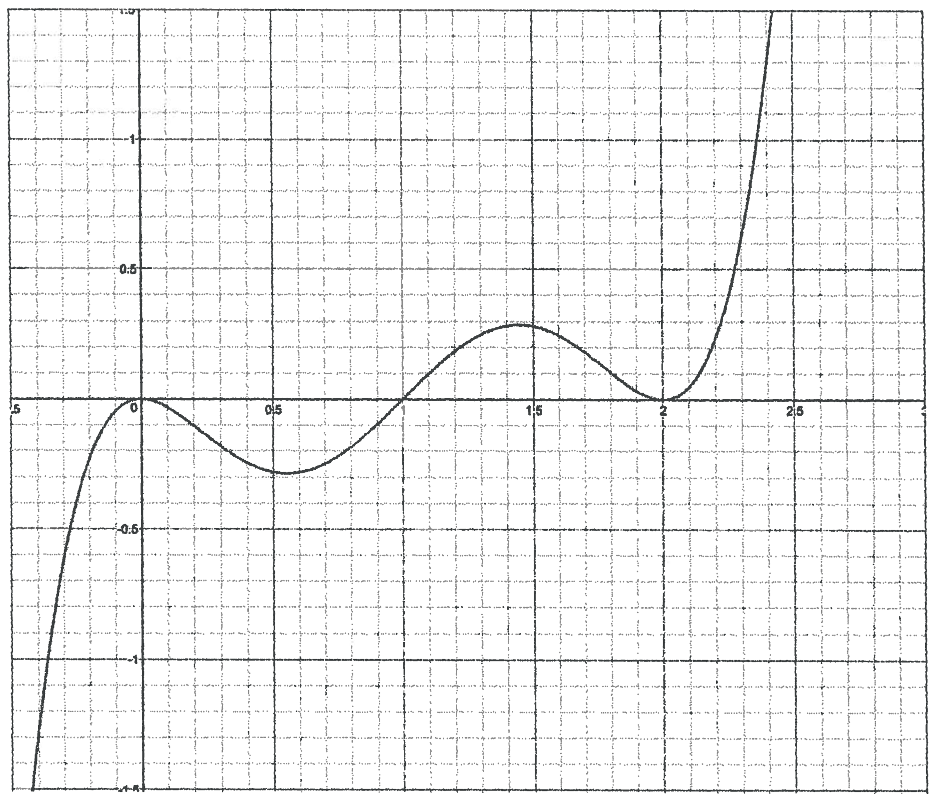
$$f''(x) = 2e^x + e^x + xe^x = 3e^x + xe^x$$

$$= e^x(3+x)$$

Set $f'' = 0$: $e^x(3+x) = 0$ when $x = -3$

HCV: $\boxed{x = -3}$

5. (5 pts) Consider a function $f(x)$ with the following graph.



(a) On what interval(s) is $f'(x) > 0$? (Approximate to the best of your ability.)

$(-\infty, 0) \cup (.55, 1.45) \cup (2, \infty)$

(b) On what intervals is f concave up? (Approximate to the best of your ability.)

$(.25, 1) \cup (1.75, \infty)$

(c) On what intervals is f' increasing? (Approximate to the best of your ability.)

same as (b)

(d) At what point(s) in $[0, 2]$ is $f(x)$ increasing the fastest? (Approximate to the best of your ability.)

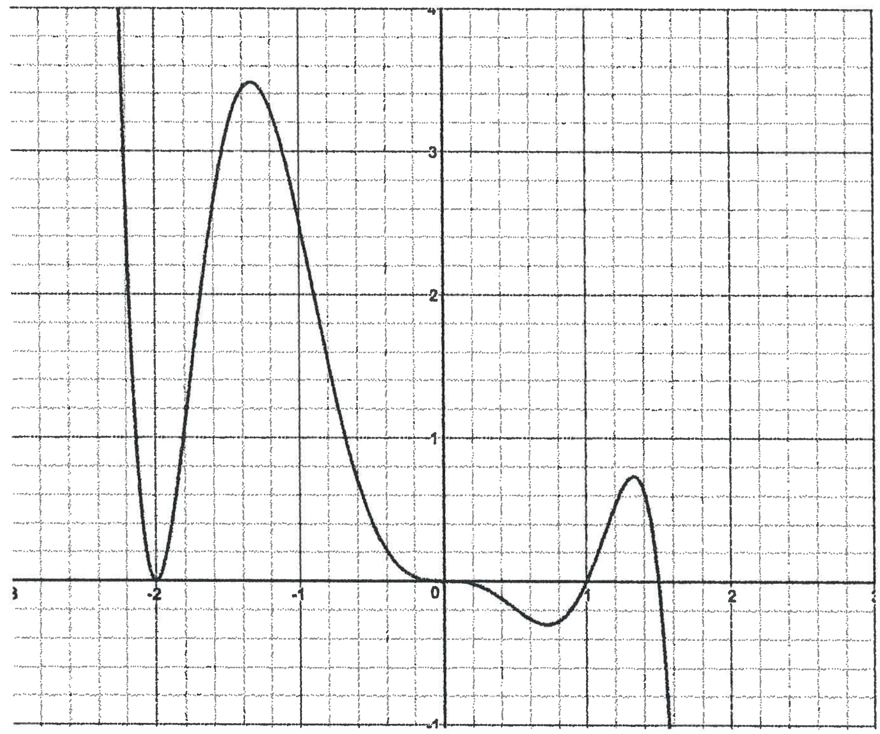
$x = 1$ (since it is an infl. pt)

(e) At what point(s) in $[0, 2]$ does $f'(x)$ have a local extremum? (Approximate to the best of your ability.)

(at any infl. pt)

$x = .25, 1, 1.75$

6. (5 pts) Consider a function $g(x)$ with the following graph of its derivative, $g'(x)$. **Repeat:** the graph below is $g'(x)$.



(a) On what interval(s) is $g(x)$ decreasing? (Approximate to the best of your ability.)

$(0,1) \cup (1.5, \infty)$

(b) On what interval(s) is $g'(x)$ concave down? (Approximate to the best of your ability.)

$(-1.8, -1.1) \cup (0, .5) \cup (1.2, \infty)$

(c) On what intervals is $g(x)$ concave up? (Approximate to the best of your ability.)

$(-2, -1.25) \cup (.8, 1.3)$

(d) At what x-value(s) does $g(x)$ have a local extremum? (Approximate to the best of your ability.)

$x = 0, 1, 1.5$

(e) At what x-value(s) does $g(x)$ have an inflection point? (Approximate to the best of your ability.)

$x = -2, -1.3, .7, 1.3$

7. (4 pts) Consider a *continuous function* $f(x)$ with the following values:

$$f(-2) = 3,\ f(-1) = 0,\ f(0) = -10,\ f(1) = -3,\ f(2) = 2,\ f(3) = 4.$$

Suppose, further, that $x = 0$, $x = 1$, and $x = 3$ are critical values of f. Give the *absolute extrema* for $f(x)$ on the set $x \in [-2, 2]$ and justify your answers.

Compare all CVs in the interval + endpts.

$$\{ \underset{3}{f(-2)},\ \underset{-10}{f(0)},\ \underset{-3}{f(1)},\ \underset{2}{f(2)} \}$$

$f(-2)$ is the absolute max, $f(0)$ is the absolute min.

8. (8 pts) Consider the function $f(x) = \dfrac{\ln(x)}{x}$ (only for $x > 0$). Compute the *local* extrema of this function, showing all work, and justifying your answer (complete sentences).

$$f'(x) = \frac{1 - \ln(x)}{x^2}$$ use FDT

set $f'(x) = 0$; $\dfrac{1 - \ln(x)}{x^2} = 0 \Leftrightarrow 1 = \ln(x)$

CV is $x = e$.

use FDT

sign f'

$+$ $-$

e

since f' changes sign at $x = e$, we know by the FDT the $(e, f(e))$ is a local max.

9. (4 pts) Consider an investment of \$100 which yields a nominal 6% interest. Compute the value of the account after 10 years, if...

   (a) Interest is compounded quarterly.

   $$A(10) = 100\left(1+\frac{.06}{4}\right)^{4\cdot 10}$$
   $$= 100(1+.015)^{40}$$
   $$\approx 181.40$$

   (b) Interest is compounded daily.

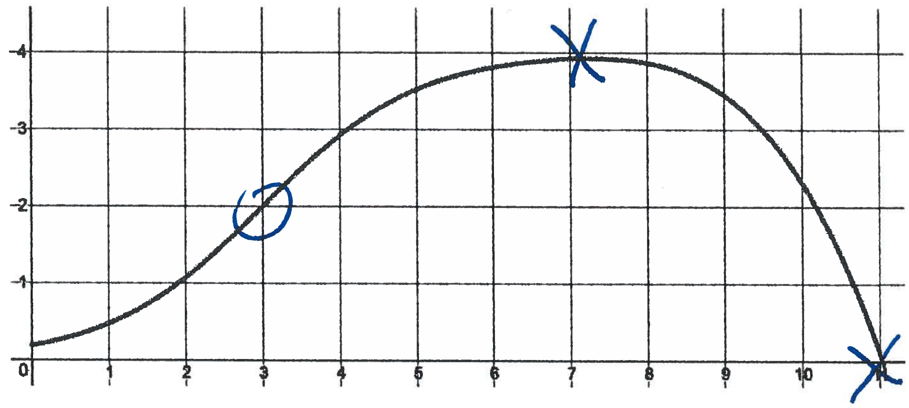
   $$A(10) = 100\left(1+\frac{.06}{365}\right)^{365\cdot 10}$$
   $$= 100(1.000164)^{3650}$$
   $$\approx 182.20$$

10. (5 pts) Consider the previous example with interest compounded continuously. How many years will it take for the investment to triple in value?

   $$3\cancel{P} = \cancel{P}e^{.06t}$$
   $$3 = e^{.06t}$$
   $$\ln(3) = .06t\ \cancel{\ln(e)}^{1}$$
   $$t = \frac{\ln(3)}{.06} \approx 18.31 \text{ years}$$

11. (7 pts) Choose **exactly one** of the following essay questions to answer:

- On the following graph, x represents a factor of production, whereas $Q(x)$ is a measure of production. Identify (by circling) the *point of diminishing returns* and explain (in four or five sentences) exactly what is happening there. Your response should include an application.



- Discuss (in four or five sentences) the *exponential growth law*. Make sure to relate it to the exponential function, and discuss a real-world application of the law/this type of growth.

(Write here.)

See notes.

[Bonus] (+5 pts) Find the inflection points of $y = e^{-x^2+2x}$. Show all work.

$$y' = (-2x+2)\,e^{-x^2+2x}$$

$$y'' = -2\cdot e^{-x^2+2x} + (-2x+2)^2\,e^{-x^2+2x}$$

$$= e^{-x^2+2x}\left[4x^2-8x+4-2\right]$$

$$= e^{-x^2+2x}\left[4x^2-8x+2\right]$$

Set $y''=0$ for HCVs

$$0 = e^{-x^2+2x}\left[4x^2-8x+2\right]$$

$$\Leftrightarrow \quad 4x^2-8x+2=0$$

$$x = \frac{8 \pm \sqrt{64-4(4)(2)}}{8} = 1 \pm \frac{1}{8}\sqrt{32}$$

$$= 1 \pm \frac{\sqrt{2}}{2}$$

Sign of f''

$+ \quad - \quad +$

$0 \quad 1 \quad 3$

$.3 \approx 1-\sqrt{2}/2 \qquad 1+\sqrt{2}/2$

Infl. pts:

$(1-\sqrt{2}/2,\ f(1-\sqrt{2}/2))$

$(1+\sqrt{2}/2,\ f(1+\sqrt{2}/2))$