1 Outline and Structure of Statement

Section 2 briefly outlines my mathematical point of view, the general class of problems I work on, and the tools and techniques I employ. In Section 3, my specific problems and areas of interest are discussed. Each major *area of interest* subsection is divided into "**Description and Past Work**", and "**Current and Future Work**". In each subsection an (abbreviated) representative reference is provided from a paper on which I am an author; full bibliographic information can be found on the last page.

2 Mathematical Point of View

My primary area of research interest is the analysis and control of *evolution equations* which describe dynamic phenomena in physics and engineering (e.g., waves, elasticity, diffusion processes, and fluid-structure interactions). Such analyses involve studying nonlinear partial differential equations (PDEs)—specifically, initial boundary value problems—and the qualitative properties of their solutions. I am also interested in the use of feedback mechanisms to drive a system to a desired state, perhaps asymptotically (*control* studies). It is typical that a viable physical model, cast within an appropriate functional setting, corresponds to a (linear or nonlinear) *strongly continuous semigroup* (or, equivalently, a *dynamical system* on a particular state space). From this point of view, we extend the study of the qualitative properties of solutions to PDEs into the realm of modern dynamical systems theory. In nonlinear systems (which are often not asymptotically stable to equilibrium), proving the existence of a set which uniformly attracts all trajectories is desirable. We seek to demonstrate that a finite dimensional, perhaps smooth, *compact attractor* exists for initial states chosen from a less smooth, infinite dimensional space. This indicates that the essential character of the non-transient dynamics is in some sense "nicer" than the initial state.

Most of my work has focused on models and phenomena which arise in the engineering literature. As such, the relationship between theoretical results for a given model, and numerically predicted or physically observed properties of the dynamics is of great interest. In the study of applied PDE systems one first pursues Hadamard well-posedness (existence, uniqueness, and continuous dependence upon initial data) of solutions. One then investigates the qualitative properties of PDE solutions. Theoretical results can be seen as predicative of the physical system, and can provide scrutiny (or verification) of the model which is supposed to describe real-world dynamics. These inquiries into the validity and correspondence of models to physical phenomenon *require active dialogues in the greater engineering and computational mathematics communities.* I emphasize that my work to date has been *phenomenon-driven*, and I have striven to interact and collaborate with engineers.

Though I consider my work to be "applied", I do not believe this is to the detriment of mathematical substance or quality. To the contrary, I believe that there is ample opportunity to study compelling and fundamentally challenging mathematical problems in the realm of physical pertinence. I also believe in looking to the engineering (and other scientific) literature for inspiration. By considering rigorous mathematical analysis, numerical simulation, and experiment *together*, one can attack a problem on multiple fronts. Moreover, studying specific classes of evolution equations naturally leads to the development, extension, and refinement of abstract tools; this is the case in both approximate and infinite dimensional studies. In working on physical PDE models, I have sought to develop novel methods of analysis which are broadly applicable in the areas of nonlinear dynamical systems and applied functional analysis.

I have frequently made use of the following tools and analytical approaches: (i) linear and nonlinear semigroups; (ii) modern dynamical systems theory; (iii) quasi-stability theory [19, 17]; (iv) boundary and interior control of PDEs [30]; (v) PDE coupled at an interface; (vi) delay equations and equations with memory.

3 Areas of Interest

My interest and experience cover various hyperbolic, parabolic, and mixed-type models, as well as coupled dynamics. I will now outline specific models, phenomena, and problems with which I have experience.

Flow-Structure Interactions and Flutter

[1] Flow-plate interactions: Well-posedness and long-time behavior, *Discr. Contin. Dyn. Syst. S*, 2014.

Today, fluid-structure interactions (FSIs) are of immense research interest across many fields; the primary goal is to understand complex coupled-response phenomena between a deforming structure and a fluid. This class of models yields a vast array of challenging problems, due to the interaction of two different dynamics across a lower dimensional interface.

Description and Past Work

One of the central issues in *aeroelasticity* concerns the onset of a systemic instability called *flutter* [13, 14, 23]. Flutter is a sustained self-excitation that occurs as a feedback between a thin structure and a surrounding fluid when the dynamic loading "destabilizes" the natural structural modes. Particular flow velocities may bring about a bifurcation of the coupled *flow-plate* system; at this point the dynamics may experience static divergence (buckling), exhibit limit cycle oscillations, or even become chaotic [24]. Flutter can occur in various applications: buildings and bridges in strong winds, panel and flap structures on vehicles, and in the human respiratory system (snoring and sleep apnea [12]). Flutter resulting from *axial flow* (long, slender bodies, with flow in the axial direction) can be achieved for *low flow velocities*, and has been studied from the point of view of energy harvesting [25]. From a design point of view, flutter cannot be overlooked, owing to sustained fatigue or large amplitude response.

As a baseline flutter model, we consider a thin, flexible plate identified (at equilibrium) with $\Omega \subset \{\mathbf{x} \in \mathbb{R}^3 : z = 0\}$; an over-body inviscid potential flow occurs in the x-direction with unperturbed velocity U ($U = 1$ normalized as Mach 1). The scalar function u models the non-dimensionalized transverse (Lagrangian) displacement of the points in $\Omega \subseteq \partial\mathbb{R}^3_+$. The scalar ϕ models the perturbation velocity potential of the flow in $\mathbb{R}^3_+$.

$$\begin{cases} (1-\gamma\Delta_{x,y})u_{tt} + \Delta^2_{x,y}u + b(u_t) + f(u) = (\partial_t + U\partial_x)\phi\big|_\Omega \ \text{ in } \Omega; \quad \text{BC}(u) \quad \text{on } \partial\Omega; \\ (\partial_t + U\partial_x)^2\phi = \Delta_{x,y,z}\phi \ \text{ in } \mathbb{R}^3_+; \quad \partial_z\phi\big|_{\mathbb{R}^2} = (\partial_t + U\partial_x)u_{\text{ext}} \ \text{ on } \partial\mathbb{R}^3_+ \end{cases} \tag{3.1}$$

The above system is obtained from the compressible Euler equations by linearizing about the steady flow state $\langle U, 0, 0\rangle$ in $\mathbb{R}^3_+$ [19] (and references therein). We can consider physical plate boundary conditions, including clamped, hinged (pinned), or clamped-free (cantilevered), as well as other fluid-plate coupling configurations. More complex fluid equations can also be considered (for instance, in the transonic regime when $U \approx 1$). The term $\gamma \geq 0$ represents rotational inertia in the filaments on the plate. Viscous/frictional structural damping in the problem can be considered in the interior or on the boundary, and viewed as naturally occurring due to the presence of the flow (via *piston-theory*, discussed below) or *imposed*; the damping is encapsulated by the term $b(u_t)$. The nonlinear terms f of interest are those arising in the theory of large deflections [29], and are discussed below. The central parameter in the problem is U. In the case $U > 1$, there is clear a loss of spatial ellipticity of the principal part of the flow operator $\partial_t^2 - \Delta + U^2\partial_x^2$. This model (and many FSI models) is not easily amenable to a functional analytic setup. The key issues are the mismatch of regularity at the interface between two interacting dynamics, and the physically required presence of ill-defined boundary traces.

Over the past 20 years, the use of Galerkin procedures [19] (and references therein) and sharp hyperbolic trace estimates [32] yielded well-posedness of the flow-plate system in (3.1) *in the presence of regularizing effects*: $\gamma > 0$ [19] or thermo-elastic effects [22]. Both additions to the model produce a smoothing of u_t. Well-posedness of physical solutions in the principal, *physical* case ($\gamma = 0$, with no additional smoothing of u_t) was open until it was recently addressed by myself and coauthors (see [2] and [40]). A non-trivial perturbation semigroup obtains in the subsonic case $0 \leq U < 1$ [40]. In the supersonic case, a viable functional analytic setup for the problem was obtained by considering a change of state variable $\phi_t \to (\partial_t + U\partial_x)\phi$. This approach leads to a splitting of the dynamics operator into a dissipative piece and a perturbative piece. With help of sharp microlocal trace estimates for the hyperbolic dynamics [30], the perturbation is shown to be energy-bounded, leading to well-posedness [2].

[2] Evolution semigroups in supersonic flow-plate interactions, *J. Diff. Equns.*, 2013.

An exciting area of recent investigation is that of flow-structure interactions in *axial flow* [13]. In this configuration, *limit cycles* can be observed at low flow velocities [13] and displacements are on the order of beam length, rather than beam thickness (as in *normal* flows). To appropriately extend the FSI model (3.1), we replace the Neumann type boundary condition for ϕ in the third line, with a *Kutta-Joukowsky* (KJ) type boundary condition written in terms of the *acceleration potential* $\psi \equiv (\partial_t + U\partial_x)\phi$ of the flow:

$$\partial_z\phi = (\partial_t + U\partial_x)u \ \text{ on } \ \Omega; \qquad \psi = 0 \ \text{ on } \ \mathbb{R}^2 \setminus \Omega. \tag{3.2}$$

Such boundary conditions are *mixed* in nature and allow for the modeling of flag or flap configurations. In the case of a clamped-free plate the structural nonlinearities are no longer in the realm of large deflections [35, 38]; see the discussion below in the **Current and Future Work** subsection. In the case of large free displacements, the linearized potential theory may be inadequate to describe the flow near the interface.

We recently studied the KJ flow condition coupled with a clamped plate in order to develop a suitable abstract flow theory (for this simplified model); the resulting papers [1, 3] prove well-posedness. The regularity properties of the dynamic *flow-to-Neumann* map are critical in permitting techniques from abstract boundary control [30], and are determined from a Zaremba elliptic problem. As in the case of supersonic dynamics [2], a trace regularity result is needed which hinges on the invertibility of an operator analogous (in two dimensions) to the finite Hilbert transform [13]. Thus, this well-posedness depends on the theory of singular integrals, and, in particular, the so called *Possio equation* [13] (addressed classically by Tricomi).

[3] Kutta-Joukowski flow conditions in flow-plate interactions: subsonic case, *Nonlinear Anal. B*, 2014.

The question of the long-time behavior of FSI solutions arises naturally; indeed the in/stability of a physical flow-plate interaction is identified with the non-transient behavior of PDE solutions. We seek to prove that the structural dynamics stabilize to a compact *global attractor*—an invariant set which uniformly attracts all bounded sets under the dynamics. Often, we can make statements about the fluid's end behavior as well.

An heuristic method for studying systems in aeroelasticity involves solving the flow equation (on the half space) via Fourier-Laplace transforms, and writing the dynamical pressure of the flow on the surface of the plate as a particular *delay potential* involving the specified flow Neumann boundary condition. This approach, made rigorous in [19] (and references therein), can be used to reduce the dynamics to a closed *plate*-only PDE system, albeit with a delay term. Another further simplification, valid for large flow velocities, assumes that the effect of the flow is comparable to that of a piston moving in an air-filled column. This so called *piston theory* allows the reduction of the full flow-plate system to a closed PDE (plate or beam equation) with non-dissipative pressure terms and no delay terms [39]:

$$u_{tt} + \Delta^2 u + f(u) = p - [u_t + U u_x] \quad \text{in} \;\; \Omega. \tag{3.3}$$

In the case of either of these simplifications mentioned above we destroy the gradient structure of the full flow-plate dynamics, disallowing powerful methods in the theory of dissipative dynamical systems. In [20], we demonstrate a *hidden compactness* property for the delay plate dynamics, and affirm that the natural damping yields a compact global attractor for the plate dynamics. The proof requires modern tools in nonlinear dynamical systems, including iterated compensated compactness techniques and *quasistability* techniques [19, 17] on the global attractor to obtain finite dimensionality and smoothness of the attractor.

In the case of subsonic flows, $U \in [0, 1)$, the flow-plate system (3.1) has a viable energy identity. If we impose additional interior structural damping in the plate model we can show trajectories for the entire system (plate *and* flow) stabilize to equilibrium points [4, 31]. The proof is based on finiteness of the dissipation integral and, a novel compact/stable decomposition of the nonlinear dynamics. This result indicates that the end behavior of the subsonic panels is in fact *static*, and is particularly exciting because it corroborates empirical findings: in the subsonic case, panels have static limiting behavior [23].

[4] Feedback stabilization of a fluttering panel in an inviscid subsonic potential flow, *SIAM J. Math. Anal.*, 2016.

Current and Future Work

My recent collaboration with aeroelasticity expert E. Dowell (Duke University) has been fruitful, resulting in recent survey articles [5, 18] that bring together engineering and mathematical points of view. As a byproduct, I have come across compelling PDE models which seem to have been neglected in the PDE literature (e.g., inextensible beam models [38], quasilinear transonic flow models [23], and generalized piston theories). These models provide challenging mathematical structures for which new theories must be developed.

I have recently explored fluid-structure models involving *viscous fluids* interacting with an elastic portion of the boundary. Such models include compressible or incompressible fluids with *prescribed ambient flows*, coupled to nonlinear shell models [21]. Such FSI models may describe the flow of blood through a large artery [33], liquid sloshing in a flexible tank, hemodynamics, or submarine locomotion.

[5] Mathematical Aeroelasticity: A Survey, *Mathem. Engin. Sci. Aerosp.*, 2016.

Much of my focus over the past few years has centered on understanding the axial flow flutter phenomenon. As discussed above, thin structures in an axial configuration exhibit very large free-end displacements, and enter stable limit cycle oscillations for low flow velocities. Predicting the onset of instability is a rather well understood linear problem [39]; however, the post flutter (nonlinear) regime is not. Specifically, the flow dynamics near the free end have only been explored experimentally and in an ad hoc way. No mathematical theory of well-posedness of solutions exists at present, due to the high degree of nonlinearity. We have recently considered a flow-beam model (which utilizes the KJ conditions above) and incorporates inextensible structural nonlinearities [38]. We have also numerically explored various *nonlinear*, higher-order pressures acting on Dowell's inextensible beam.

Recently, I have incorporated a numerical point of view into my work by utilizing finite element/difference techniques, as well as *modal analysis* [39] applied to non-dissipative, nonlinear structures. This is joint work with my colleagues J. Howell (College of Charleston) and D. Toundykov (University of Nebraska–Lincoln), which marries recent work on exponential attractors for the nonlinear piston-theoretic plate, in the presence of large damping, with a robust set of numerical simulations [6]. This work has also critically involved my graduate and undergraduate students. One of our primary goals is to understand the qualitative effects of inextensible and extensible nonlinearities for the unstable (fluttering) regime in the cantilevered (axial) configuration.

[6] Quasi-stability and Exponential Attractors..., *Evolution Equns. Contr. Theory*, 2016.

Nonlinear Elasticity

Independent of FSIs, the study of nonlinear plates and shells has attracted great attention in the mathematical and engineering communities over the last 50 years [14]. I am interested in models of the large deflections of thin (2D) structures: the full von Karman shell model (which assumes finite elasticity, and accounts for in-plane and out-of-plane dynamics); the scalar von Karman model [29] (a nonlocal simplification, arrived at by neglecting in-plane accelerations); and Berger's simplification [8] (arrived at by assuming the second strain invariant is negligible). The scalar equations discussed here model the transverse deflection of the central plane of the plate. Though we do not give a full description of the scalar von Karman equation (referred to as f_V), we suffice to say that it is nonlocal, of cubic type on $H^2(\Omega)$. Well-posedness and long-time behavior investigations for von Karman plates have been discussed extensively in recent years. The key breakthrough in the modern study of von Karman equations came only recently in establishing sharp estimates on the von Karman bracket from $H^2(\Omega)$ into the Lizorkin space $F^0_{1,2}(\Omega)$ [19]; this leads to the so called *sharp regularity* of the Airy stress function—which is critical for well-posedness and long-time behavior studies.

The so called Berger nonlinearity is given by $f_B(u) = (b_1 - b_2||\nabla u||^2_{L_2(\Omega)})\Delta u$. The parameter $b_1 \in \mathbb{R}$ measures in-plane tension (or compression) at equilibrium, and $b_2 \geq 0$ measures the strength of the nonlinear effect of stretching on bending. As with f_V, we note that the f_B nonlinearity is nonlocal and of polynomial type (on the energy space). We remark that, when the displacement of the boundary is fixed, the Berger simplification can be made without greatly affecting the accuracy of the model [34].

Description and Past Work

Motivated by the complications of boundary damping, one can consider localized interior damping near the the boundary of the plate. This type of damping, referred to as *geometrically constrained interior damping* (GCID), takes the form of a velocity feedback, supported in a prescribed subset of the elastic domain. GCID has been utilized for many hyperbolic models, and has produced a bounty of results for the wave equation ([41], for instance). However, at the time of [7], no analyses were available in the context of global attractors for nonlinear plates. The effectiveness of GCID is contingent upon the "propagation of dissipation" from the boundary layer throughout the interior of the structure. This itself depends upon a *unique continuation* property, which requires that the nonlinear dynamics interact with the damping region in a desirable way. GCID can also be demanding to implement, as energy methods introduce critical commutator terms which must be controlled (at the energy level) in an observability type estimate. In [7] we show that the fully *nonlinear* damping with essential support in an arbitrarily small layer near the boundary provides, not only the existence of compact attractors, but also C^∞ spatial smoothness of solutions and finite dimensionality of the attractor. Our result is valid for *all types of boundary conditions* (including the third order *free plate boundary conditions* which do not comply with the Lopatinski conditions [32]).

[7] Smooth attractors of finite dimension for von Karman evolutions..., *J. Diff. Equns*, 2013.

A systematic study of von Karman (vK) plates with interior and boundary damping was undertaken over the last 20 years [19]. Interesting issues arise in comparing mathematical properties of f_B and f_V, as well as their physical applicability. In the engineering literature it is not clear under which circumstances the Berger *ad hoc* simplification of vK dynamics is valid [34]. We pursued a line of investigation of the Berger plate with inhomogeneous boundary conditions [8], primarily for comparison with the results on the scalar vK equation [19]. For free boundary conditions, well-posedness requires highly nonlinear damping on the boundary, which is not necessary in the analogous vK case; this indicates the breakdown of the Berger model in this configuration. In the case of boundary damping, acting through the hinged configuration, we were able to show stronger results than the analogous work on the vK plate with the same boundary damping. In [8, 11], we show that nonlinear boundary damping (with other mild, standard assumptions) is sufficient to obtain a compact global attractor [8]; moreover, the damping need only be active on a portion of the boundary, and also that the aforementioned global attractor is in fact *smoother* than the state space, as well as finite dimensional [11].

[8] Qualitative Results on the Dynamics of A Berger Plate..., *Nonlin. Anal. B*, 2016.

Current and Future Work

The full von Karman (FvK) equations [28, 29] mentioned above are vectorial equations which describe the nonlinear coupled dynamics of in-plane and transverse motions of a thin plate or shell. Use of these equations presents an additional degree of accuracy in modeling. When we do not simplify to the scalar von Karman equation, we retain the *local* character of the nonlinearity, which can be advantageous. Yet, in the simpler, scalar case, many deep results are known about the structure of the nonlinearity which are not available for the FvK dynamics (e.g., locally Lipschitz property). As such, existence and uniqueness of energy type solutions has been a nontrivial issue. Well-posedness results are available, however, these results depend heavily on the presence of the regularizing rotational inertia term ($\gamma > 0$) and the proofs are highly non-trivial. In the primary case of physical interest ($\gamma = 0$), uniqueness of weak solutions is at issue [28]. In considering the more general class of flow-structure models described above (e.g., axial flow, viscous fluids) *the FvK equations may be necessary* to account for the highly nonlinear, in-plane/transverse coupled dynamics. Indeed, recent work concerning fluid-structure interactions (involving blood flow [33], as well as the *sloshing* phenomenon), consider the FvK equations in FSIs [21]. These works do not consider the FvK shell model in the clamped-free configuration, nor in the presence of a potential flow (i.e., in the study of flutter). We intend to incorporate the FvK equations into numerical and analytical work involving these FSI considerations.

We mention the inextensible model in [38], utilized in the cantilevered configuration with large free-end displacements. Inextensible models are distinct from those based on the von Karman hypotheses, since in the case of a clamped-free or free-free beam/plate the effect of stretching on bending is minimal. In this case, the nonlinear *inextensibility* condition [35] leads to nonlinear stiffness and inertia effects. Independent of any fluid, developing a suitable functional setup for solutions is a principal goal.

Porous Media

Considering certain types of fluid flow through a porous medium leads to so called *porous medium equations*, which are nonlinear/multivalued parabolic evolutions, not unrelated to other parabolic problems (e.g., the Stefan free boundary problem). When a porous medium is considered to have elastic properties—which interact with a slightly compressible fluid—this leads to a class of poro-elasticity models (e.g., the *Biot* model). Deformation of a porous solid affects the fluid flow, and fluid pressures influence the mechanical behavior of that structure. The classical models of this interaction were developed in soil science [37], but have recently been considered in the context of mathematical biology [10].

Description and Past Work

Methane hydrates are ice-like formations which occur in shallow seafloor sediment under specific pressure-temperature conditions. Hydrates consist of methane particles which are "trapped" in a lattice of water molecules; they are of recent interest due to: the impact of methane as a greenhouse gas, the instability of hydrates in seafloor drilling, and the possibility of hydrates as an energy source. The formation/dissociation of hydrates in nature is poorly understood, and locating hydrate regions based on seafloor measurements is challenging.

Recent work provided an abstract (semigroup) framework for a nonlinear/multivalued parabolic evolution modeling the phase transitions of methane hydrates [26] due to *diffusion*. Empirical literature in the past few years indicates that methane *transport* is also key to hydrate formation; in [9] we produced a refinement of the model in [26] taking into account advective methane flux. The model is arrived at from mass conservation and a thermodynamic principles (which yield a *nonlinear complementarity condition*). Upon simplification, it is seen to be a porous medium equation with heterogeneous constraints and advection (the latter making the problem non self-adjoint). The nonlinear character of the hydrate problem is reminiscent of the classical Stefan problem, though the multivalued graphs at play are parametrized in space:

$$\partial_t \beta(x, v(x,t)) - \mathrm{div}_x \left(\mathbf{q} v(x,t) - \nabla_x v(x,t) \right) \ni f(x). \tag{3.4}$$

The family $\beta(x, \cdot)$ is a multivalued family of monotone graphs, and $\mathbf{q}$ is a prescribed mass flux. The abstract framework here necessitates seeking L^1 in space solutions, which excludes the classic Kato generation theorem. The problem is resolved via semigroups in the context of nonlinear/multivalued operators on Banach spaces (including L^1 in space); the stationary problem is considered via an extension of the classic work in [15] for nonlinear elliptic equations. By considering a weaker notion of solution (Crandall-Liggett-Benilan [36]) than in prior work [26]: (i) well-posedness of the associated Cauchy problem is obtained, (ii) a maximum principle is demonstrated (which guarantees that physical data corresponds to bounded solutions), and (iii) complementary numerical results are presented [9].

[9] Advection of methane in the hydrate zone: model, analysis and examples. ***Math. Meth. Appl. Sciences*****, 2015.**

Much work has been done on classic systems of *poro-elasticity.* The principal model of consideration is the so called Biot model [37] (and references therein), which couples an equation of static linear elasticity for a porous matrix to Darcy's law for the fluid pressure. The elasticity equation is driven by the pressure gradient of the fluid, and the diffusion equation relates dilation of the porous matrix to dynamic convection. In the case of fully linear dynamics (with Darcy's law assumed) the system is elliptic/parabolic, and is well understood. An elegant semigroup theory for implicit evolution equations of the form $\partial_t[Bp] + Ap \ni 0$ (an abstraction of the Biot system) has been developed [37]. This theory, and corresponding applied analyses, consider the general case for linear poro-elasticity (Biot's classical model), as well as nonlinear extensions which are pertinent in geoscience. In all cases, the system can be reduced to a (possibly nonlinear) implicit evolution of the form above.

Our recent work, driven by the application of modeling the dynamics of the human eye (specifically, the lamina cribosa—LC), set out to develop the mathematical theory of a particular nonlinear coupling in the Biot model, as well as visco-elastic effects. The well-posedness of this nonlinear Biot system is under consideration, and both quantitative and qualitative properties of solutions, and the effects of individual terms in the systems, is being studied. Specifically, in [10], we considered a nonlinear elliptic/parabolic system of PDEs that models fluid flow through poro-visco-elastic material. The ability of the fluid to flow within the solid is described by a permeability tensor which varies nonlinearly with the dilation of the matrix (the key point of departure from [37]). Though a similar model was considered in [16], it did not include viscous effects (perhaps the principal term in our analysis), and the boundary conditions in [16] are simplified to an extent that the model is no longer viable for our biological (LC) application. (The physical *mixed* type boundary conditions we consider are a chief source of difficulty in the analysis.) The existence of weak solutions in bounded domains is shown in [10] with physical boundary conditions using a variant of Rothe's method. We also account for non-zero volumetric and boundary sources, with a principal aim of investigating the influence of viscoelasticity on the qualitative properties solution.

Our analysis shows that different time regularity requirements are needed for the volumetric and boundary sources, depending on the presence of visco-elasticity. Theoretical results are further investigated in [10] via numerical simulations, and experiments corroborate the theoretical findings: when data are appropriately regular, numerically simulated solutions satisfy predicted energy estimates. Simulations also show that, in the purely elastic case, the Darcy velocity and the related fluid energy may become unbounded if the data do not enjoy the requisite time regularity. Finally, our results are interpreted in the context of rapid pressure changes in LC, and the connection between these biomechanics and the development of glaucoma due to the deterioration of visco-elasticity in the porous tissue of the LC.

[10] Analysis of nonlinear poro-elastic and poro-visco-elastic models. *Arch. Rational Mech. Anal.*, 2016.

Current and Future Work

With existence of solutions obtained in [10], the natural questions of their uniqueness and (additional) regularity arise. We conjecture that, at least in the visco-elastic case, the dynamics preserve spatial regularity. And thus, if a particular degree of smoothness is needed (for instance, to show uniqueness of solutions), this may be achieved by considering smooth initial data. Due to the physical requirement of mixed boundary conditions (with possible intersection of Dirichlet and Neumann portions), and the nonlinear nature of the coupling, classical regularity analyses of parabolic equations do not apply. A general regularity theory is currently being developed for solutions which considers arbitrary time-dependent coefficients, and sharp elliptic estimates for the mixed boundary conditions. This is a first step in applying a general approach by Kato for evolutions (described below). At present, it appears that there is no complete regularity theory for the general Biot model with time-and-space-dependent permeability/porosity.

From a priori estimates, and mild assumptions on the nonlinear coupling, uniqueness of solutions follows from $\nabla p(t) \in L^\infty(\Omega)$ [16]. In light of the previous paragraph, one motivation for a regularity analysis is, then, to see if smooth data is sufficient for uniqueness. This can be addressed by repeatedly differentiating the nonlinear equations, and exploiting the specific structure of the coupling in order to close estimates which bound the dynamics at the level of finite energy solutions.

Classic work by Kato [27] introduced a technique for obtaining well-posedness of parabolic and hyperbolic evolution equations of the form $y' = A(y(t))y$. The general approach involves fixed point arguments applied to arbitrary evolution equations of the form $y' = A(f(t))y$, along with a collection of "uniform" a priori estimates. Implementing this abstract theory for nonlinear poro-visco-elasticity is a goal, emanating from the regularity analysis described above. Such an analysis would be beneficial (even for simplified equations), in that it would provide uniqueness and an appropriate notion of continuous data dependence.

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