3/30/2017

Quiz 4 Solutions

MATH 105, Spring 2017

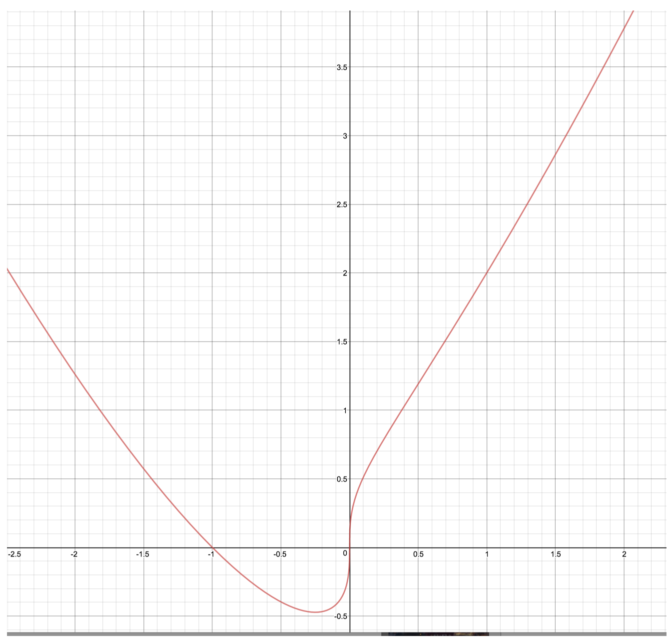
Name (Please print):____

For this quiz you may refer to the following information:

Let $f(x) = x^{1/3}(x+1) = x^{4/3} + x^{1/3}$. Then, as per class:

$$f'(x) = \frac{4}{3}x^{1/3} + \frac{1}{3}x^{-2/3} = \frac{1}{3x^{2/3}}(4x+1)$$

$$f''(x) = \frac{4}{9}x^{-2/3} - \frac{2}{9}x^{-5/3} = \frac{2}{9x^{5/3}}(2x-1)$$



1. (1 pt) List the (exact) critical value(s) of $f(x)$.

   The critical values of $f(x)$ occur for x-values in the domain of f such that $f'(x) = 0$ or DNE. In this case, from above, it is clear that $f'(-1/4) = 0$ and $f'(0)$ DNE. Thus $x = -1/4, 0$ are the critical values of f, and thus the candidates for local extrema. To determine if those x-values are extrema, we will have to *test* them (below).

2. (1 pt) List the (exact) hypercritical values of $f(x)$.

   The hypercritical values of $f(x)$ occur for x-values in the domain of f such that $f''(x) = 0$ or DNE. In this case, from above, it is clear that $f''(1/2) = 0$ and $f''(0)$ DNE. Thus $x = 0, 1/2$ are the hypercritical values of f, and thus the candidates for inflection points. To determine if those x-values are inflection points for f, we will have to *test* them (below).

3. (5 pts) Find the (exact) x-values of the local extrema of $f(x)$ using *either* the First Derivative Test or the Second Derivative Test. Show your work and justify your answers.

To use the first derivative test, we need to test the *sign of* $f'(x)$ on either side of each CV; typically this is done with a number line (which we omit here, since I can't easily draw it). Let us check the sign of f' at $x = -1$, $-1/8$, and 1.

$$\begin{aligned} f'(-1) &= (+)(-) = - \\ f'(-1/8) &= (+)(+) = + \\ f'(1) &= (+)(+) = +. \end{aligned}$$

Since the sign of f' changes at the critical value $x = -1/4$, we know by the FDT that $(-1/4, f(-1/4))$ is a local extremum (a local min), and $x = 0$ is not an extremum.

If we were to test the critical values using the *second derivative test*, we'd note that $f''(-1/4) = (-)(-) = +$; this means that f is concave up at the CV $x = -1/4$, and by the second derivative test, we know that $x = -1/4$ corresponds to a local minimum. Note that $f''(0)$ is undefined, and thus the SDT doesn't apply here.

4. (5 pts) Find the (exact) x-values of the inflection point(s) of $f(x)$. Show your work and justify your answers.

   For points of inflection, we are looking for places where $f''(x)$ changes sign. This can happen at HCV, which we know to be $x = 0, 1/2$. Thus we would make a sign chart for $f''(x)$, which we omit here. Let's test $f''(-1)$, $f''(1/4)$, $f''(1)$.

$$\begin{aligned} f''(-1) &= (-)(-) = + \\ f''(1/4) &= (+)(-) = - \\ f''(1) &= (+)(+) = +. \end{aligned}$$

   Since the concavity of f changes at each of the hypercritical values $x = 0, 1/2$, we know that both $(0, f(0))$ and $(1/2, f(1/2))$ are inflection points of $f(x)$.

5. (3 pts) Find the (exact) x-values of the local extrema of $f'(x)$. Show your work and justify your answers.

   To do this, we'd look at the first derivative of the function we are trying to optimize, namely $[f']'$. Since this is equal to $f''(x)$, we are looking for sign changes of f'', which occur precisely where we determined in the previous step (same sign chart). Thus we are tempted to say that all inflection points of f occur at the same places as the local extrema of f'. However, since $f'(0)$ is undefined, this means that 0 is not in the domain of f', and so cannot be an extremum. Thus we have that $(1/2, f'(1/2))$ is a local extremum (a local min) for $f'(x)$.