Quiz 3(ish) Solutions
MATH 105, Spring 2017

Name (Please print):__________________________________________________________

Answer the following questions, showing all work when appropriate.

1. (7 pts) Consider the function \( f(x) \) given by the graph below:

(a) Give the intervals upon which \( f \) is increasing.

\( f \) is increasing when the slopes of the tangent lines are positive; one can also say \( f \) is increasing whenever the graph is “getting bigger” as we move from left to right.

Those intervals are: \((-\infty, -6) \cup (-2, 0) \cup (2, \infty)\)

(b) Give the critical \( x \)-values for \( f \).

Critical values for \( f \) are places in the domain where \( f'(x) = 0 \) or \( DNE \), which is to say the tangent line has zero slope, vertical slope, or no well-defined slope.

These appear to be \( x = -6, -4, -2, 0, 2 \)

(c) Give the local minima of \( f \) as ordered pairs.

One can find these by inspection.

\((-2, f(-2)), (2, f(2))\)

(d) Give the absolute extrema for \( f \) on \([1, 5]\).

These are simply the highest and lowest \( y \)-values on the interval \([1, 5]\).

The absolute max on this interval is \((5, f(5))\), which is an endpoint. The absolute min on this interval is clearly \((2, f(2))\), which is a local min as well.
(e) Give the intervals upon which $f$ is concave down.

$f$ is concave down whenever $f''(x) < 0$, which is equivalent to saying whenever $f$ looks like a “cap”, and also equivalent to when $f'$ is decreasing.

This occurs on the intervals: $(-\infty, -5) \cup (-4, -3) \cup (-1, 2) \cup (2, 4)$.

(f) Give the intervals upon which $f'$ is decreasing.

This is the same question as the previous, since $f'$ is decreasing when $[f']' < 0 \iff f'' < 0$. Thus the answer is identical to the previous one.

(g) Give the $x$-values of inflection points of $f$.

Inflection points of $f$ occur whenever the concavity of $f$ changes. This can be seen by inspection.

$x = -4, -3, -1, 4$.

2. (3 pts) Let $g(x) = x^4 - 24x^2 + 3x + 1$. Find the intervals where $g$ is concave down. Show all work. Give answers as ordered pairs.

We note that $g'(x) = 4x^3 - 24x^2 + 3$, and so $g''(x) = 12x^2 - 48x = 12(x^2 - 4)$. The hypercritical values are (in this example) when $g''(x) = 0$. It is clear that this occurs when $x = \pm 2$, and thus these are the HCV. I am omitting the sign chart here, but we test values of $g''$ on either side of the HCVs:

$g''(-3) = +, \ g''(0) = -, \ g''(3) = +$.

Thus, since $g''$ changes sign at the HCVs, each HCV corresponds to an inflection point: $(2, f(2))$ and $(-2, f(-2))$ are both inflection points of $f$, since the concavity changes there.

(Bonus) (+2 pts) Let $S(q)$ be a sales function for the amount of money invested $q$. It is understood that the point of diminishing returns is the inflection point in the graph of $S(q)$.

Draw and label a graph below illustrating this inflection point, and explain what this point means in terms of the money invested, $q$, and the behavior of the sales, $S(q)$.

The inflection point occurs at $x = 3$, where the concavity changes. This is the place (on this curve) where the first derivative $S'(q)$ is maximized, representing the maximal per unit return on investment—which is to say one is getting the “best bang for your buck” in terms of how much of $S$ you are increasing per unit of $q$ you are adding. Mathematically, at this point you have maximized the slope of the tangent line $\iff$ you are at a local max for $S' \iff$ you are at an inflection point for $S$. 