3/23/2017

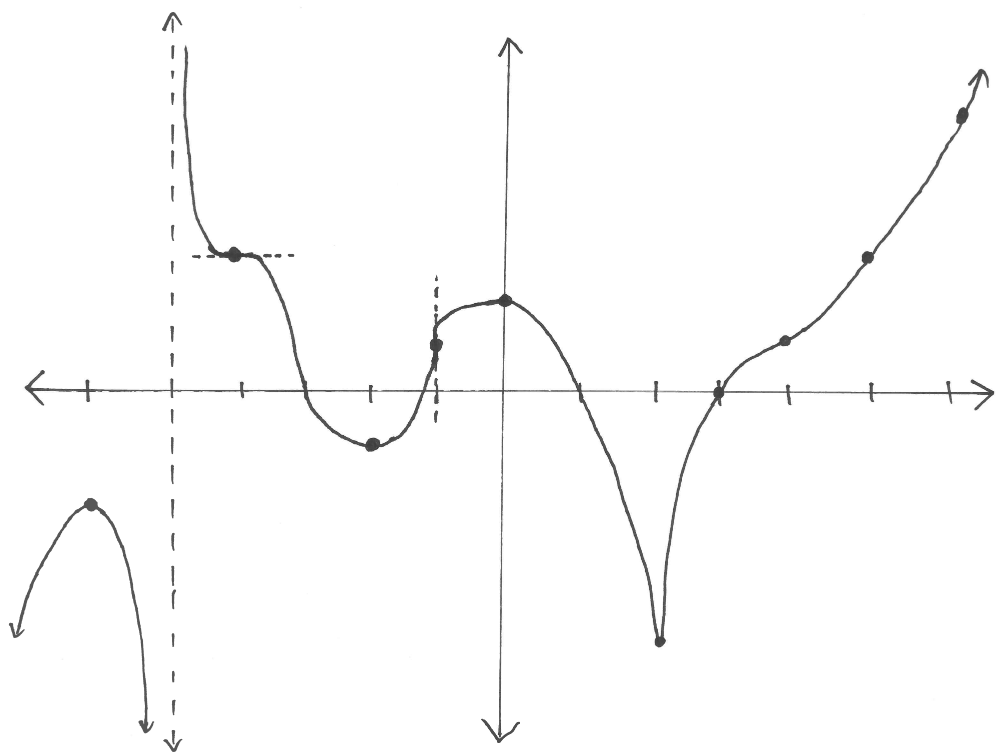
Quiz 3(ish) Solutions

MATH 105, Spring 2017

Name (Please print):____

Answer the following questions, **showing all work when appropriate**.

1. (7 pts) Consider the function $f(x)$ given by the graph below:



(a) Give the intervals upon which f is increasing.

f is increasing when the slopes of the tangent lines are positive; one can also say f is increasing whenever the graph is "getting bigger" as we move from left to right.

Those intervals are: $(-\infty, -6) \cup (-2, 0) \cup (2, \infty)$

(b) Give the critical x-values for f.

Critical values for f are places in the domain where $f'(x) = 0$ or DNE, which is to say the tangent line has zero slope, vertical slope, or no well-defined slope.

These appear to be $x = -6, -4, -2, 0, 2$

(c) Give the local minima of f as ordered pairs.

One can find these by inspection.

$(-2, f(-2)), (2, f(2))$

(d) Give the absolute extrema for f on $[1, 5]$.

These are simply the highest and lowest y-values on the interval $[1, 5]$.

The absolute max on this interval is $(5, f(5))$, which is an endpoint. The absolute min on this interval is clearly $(2, f(2))$, which is a local min as well.

(e) Give the intervals upon which f is concave down.

f is concave down whenever $f''(x) < 0$, which is equivalent to saying whenever f looks like a "cap", and also equivalent to when f' is decreasing.

This occurs on the intervals: $(-\infty, -5) \cup (-4, -3) \cup (-1, 2) \cup (2, 4)$.

(f) Give the intervals upon which f' is decreasing.

This is the same question as the previous, since f' is decreasing when $[f']' < 0 \iff f'' < 0$. Thus the answer is identical to the previous one.

(g) Give the x-values of inflection points of f.

Inflection points of f occur whenever the concavity of f changes. This can be seen by inspection.

$x = -4, -3, -1, 4$.

2. (3 pts) Let $g(x) = x^4 - 24x^2 + 3x + 1$. Find the intervals where g is concave down. Show all work. Give answers as ordered pairs.

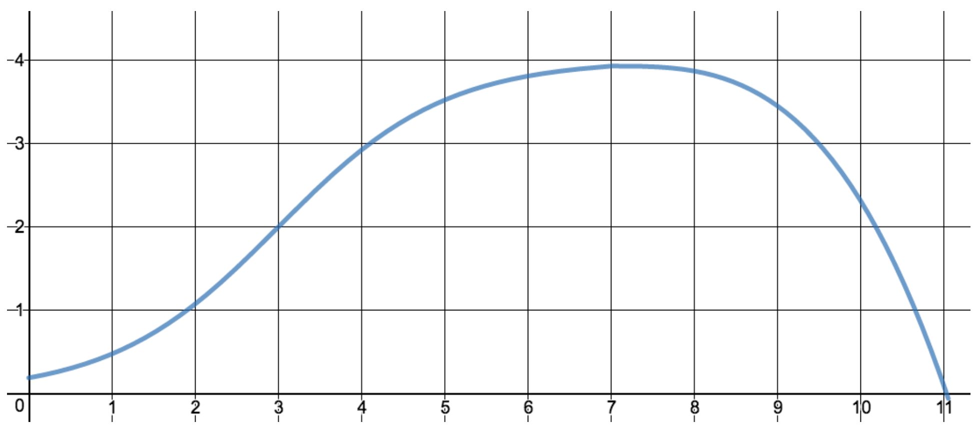
We note that $g'(x) = 4x^3 - 24x^2 + 3$, and so $g''(x) = 12x^2 - 48x = 12(x^2 - 4)$. The hypercritical values are (in this example) when $g''(x) = 0$. It is clear that this occurs when $x = \pm 2$, and thus these are the HCV. I am omitting the sign chart here, but we test values of g'' on either side of the HCVs:

$$g''(-3) = +, \; g''(0) = -, \quad g''(3) = +.$$

Thus, since g'' changes sign at the HCVs, each HCV corresponds to an inflection point: $(2, f(2))$ and $(-2, f(-2))$ are both inflection points of f, since the concavity changes there.

(Bonus) (+2 pts) Let $S(q)$ be a sales function for the amount of money invested q. It is understood that the *point of diminishing returns* is the inflection point in the graph of $S(q)$.

Draw and label a graph below illustrating this inflection point, and explain what this point means in terms of the money invested, q, and the behavior of the sales, $S(q)$.



The inflection point occurs at $x = 3$, where the concavity changes. This is the place (on this curve) where the first derivative $S'(q)$ is maximized, representing the maximal per unit return on investment—which is to say one is getting the "best bang for your buck" in terms of how much of S you are increasing per unit of q you are adding. Mathematically, at this point you have maximized the slope of the tangent line $\iff$ you are at a local max for S' $\iff$ you are at an inflection point for S.