1/31/2017

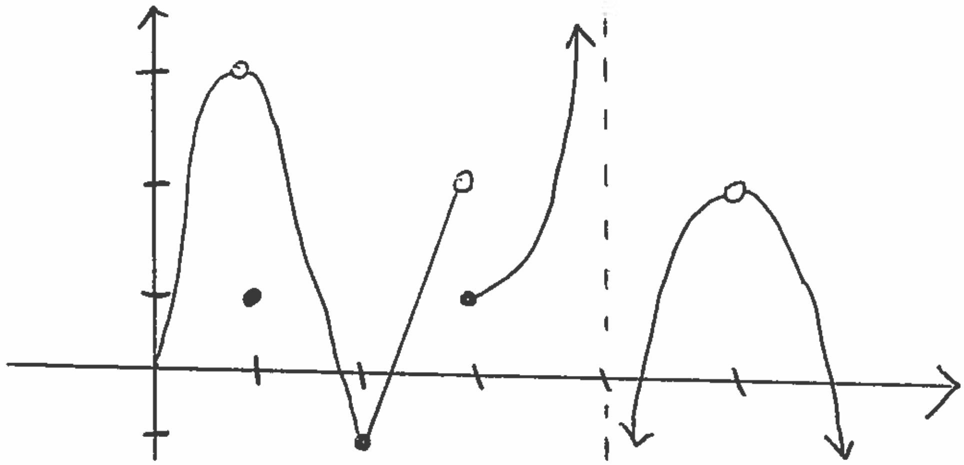
Quiz 1

MATH 105, Spring 2016

Name (Please print): KEY

Answer the following questions, showing all work where appropriate.

1. (5 pts) Consider the function $f(x)$ given by the graph below:



Give the following limits, and if the limit does not exist, please write "DNE".

$\lim_{x\to 1} f(x) = 3$ (although $f(1) = 1$)

$\lim_{x\to 2} f(x) = -1$ (and $f(2) = -1$ as well)

$\lim_{x\to 3} f(x) =$ DNE (different behavior on either side)

$\lim_{x\to 4} f(x) =$ DNE (vertical asymptote)

$\lim_{x\to 5} f(x) = 2$ (even though $f(5)$ DNE)

2. (5 pts) Compute the following limits, showing all work and giving explanation if necessary.

(a) $\lim_{x \to 2}(x - 7) =$ $2 - 7 = -5$ (just plug in)

(b) $\lim_{x \to 2} \frac{x+2}{x-2} =$ DNE since $\frac{\#}{0}$

(c) $\lim_{x \to 2} \frac{x-2}{x+2} =$ $\frac{0}{4} = 0$ (just plug in)

(d) $\lim_{x \to 2} \frac{x^2 - x - 2}{x - 2} =$ $\lim_{x \to 2} \frac{\cancel{(x-2)}(x+1)}{\cancel{(x-2)}} = \lim_{x \to 2}(x+1) = 3$

since "lim" ignores $\underline{x=2}$

(e) $\lim_{x \to -2} \frac{x^2 - x - 2}{x - 2} =$ $\frac{4 - (-2) - 2}{-2-2} = \frac{4}{-4} = -1$

3. (3 pts) Let $f(x) = x^2$. Note that: $\dfrac{f(x+h)-f(x)}{h} = \dfrac{2xh+h^2}{h}$.

Compute

$$\lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{2xh+h^2}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{h}}{\cancel{h}}(2x+h)$$

since "lim" ignores $h=0$

$$= \lim_{h\to 0} (2x+h)$$

$$= 2x+0$$

just plug in, now

$$= \boxed{2x}$$

4. (2 pts) For a function $f(x)$, explain the difference between the notions of *instantaneous rate of change*, and *average rate of change*.

(Your answer should be about three *complete sentences*.)

The a.r.o.c. of f measures change across an interval. The i.r.o.c. measures a rate of change at a particular instant (or point).

5. (5 pts) You will open a factory to produce widgets and sell them from a small store nearby. These widgets cost \$5 to produce (each), and your fixed (overhead) cost is \$200. You pay a firm to help you price the widgets, and they inform you that if x is the number of widgets, then the demand function $p = D(x)$ is a linear function: at $x = 5$, $p = \$30$, and at $x = 25$, $p = \$10$.

Write down the *profit* function (P as a function of x) and simplify to the standard (unfactored) form for a polynomial.

To find $p = D(x)$ create a line from $(5,30)$ + $(25,10)$. $m = \frac{30-10}{5-25} = -1$

$p - 10 = -1(x-25) \Rightarrow p = D(x) = -x + 35$

Thus $R(x) = xp = xD(x) = x(-x+35) = -x^2+35x$

$C(x) = 5x+200$

So $P(x) = R(x) - C(x)$

$= -x^2+35x - [5x+200] = \boxed{-x^2+30x-200}$

(Bonus) (+3 pts) Write down the break-even points for the profit function you found above.

$P(x) = -x^2+30x-200$

$= -1(x^2-30x+200)$

$= -1(x-10)(x-20)$

BEP are zeros of $P(x)$:

$\boxed{x=10 \ + \ x=20}$ units