Math 120 Enrichment 1: Calculus Changes Everything

The following is a description of an *enrichment* activity that will be worth about 7.5% of your final grade. You will be scored a zero, five, nine or ten, depending on the quality of what you turn in. **This is a short-to-medium written assignment which will be strictly graded**, based on the following criteria:

- Quality of presentation and writing: clarity, use of appropriate English syntax and grammar;
- Correctness: are the mathematics correct? Are they symbols and notation consistent and appropriate?
- Content and flow: does the paper actually make sense and does it read well? Is it compelling?
- Depth of research: how much work went into investigating the topic and formulating the paper? Does it appear that the author understands the mathematical concepts?

Note: this will be graded strictly. If it is clear that little time was spent, or in-depth thought was not given to the research and exposition, then a zero (or five) will likely be given. Do not expect to score highly if you only put in an hour or two of work.

This activity will be due by Monday, Oct. 10th in class.

The assignment is based on the fact that, within mathematics and science, exposition does matter! Being able to clearly communicate mathematical ideas is as important as understanding the ideas themselves. *Getting the "right" answer doesn't matter if you can't convey your work to another*! Additionally, developing the skills to research, internalize, and convey difficult concepts is one of the most important aspects of a mathematics course. Thus, this enrichment is based on doing mathematical research, internalizing it, and then conveying it to an audience (me, and your classmates).

Prompt: Write a 6–8 page (double-spaced, not including the references) paper which fleshes out any of the topics described below. In particular, I want you to read about the topic (from various legitimate sources), develop a baseline understanding, and then continue refining your understanding of the notation, theorems, and computations involved. I want you to demonstrate to me that you deeply understand the use/application of calculus concepts in the topic of your choosing. Beyond this, you may be creative with your exposition. This paper must find a balance between exposition and mathematics, i.e., it cannot *just* be mathematics, nor can it *just* be explanation/history.

- *Newton v. Leibniz*—For this prompt you will explore the (in)famous controversy about the invention of "the calculus". In the modern era we attribute the (co)invention of calculus to both Gottfried Wilhelm Leibniz and Sir Isaac Newton, however, it hasn't always been this egalitarian. During the era of Newton and Leibniz there was an intense rivalry, reality-TV type behavior, and hundreds of years of lasting disagreement that had profound impacts on

European mathematics. Your job is to explore: (i) the history of the invention of the calculus, (ii) the rivalry between the two men (and their respective schools), (iii) how their ideas were similar/different (with a particular emphasis on their notation), (iv) the the accusations and controversy which spread throughout Europe, (v) the legacy of the feud in mathematics (lasting even until today), and, finally, (vi) your educated opinion about who *actually* invented calculus.

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- *Neither: Before Newton and Leibniz*—But wait a minute (see the above prompt). Is it possible that calculus was invented before either of them? Sure, Newton and Leibniz invented modern calculus, but is it possible that the key ideas of the calculus were around far before the 17th century? In this prompt you can explore ancient civilizations and/or thinkers for which the invention of calculus (differential or integral) could be attributed. There are many ways to go with this prompt, so I suggest reading around and choosing a civilization or a thinker, and then: (i) give a historical context and description of the thinker/civilization, (ii) an account of what problems they were trying to solve, (iii) the method or technique which is a precursor to a modern calculus idea, and, finally, (iv) an argument as to why the *invention of calclulus* (or at least one of differential or integral calculus) should be attributed to neither Newton nor Leibniz. Some starting points: ancient Egypt (and volume problems), ancient India (and trigonometry), ancient Greece, including possibly: Archimedes, Eudoxus, Democritus and/or Zeno; early modern era Chinese, including: Zu Chongzhi and/or Liu Hui. This prompt is a little more "open" than the former, in that you have a lot of freedom of choice of content as long as you hit the bullet points above.

  Active Link 1, Active Link 2

- *Fundamental Motivating Problems of the Calculus*—In this prompt you have freedom to explore, in more specificity, a problem (or the problems) which inspired calculus. Unlike the above prompts, which focuses on the invention/inventors of calculus, in this prompt you will choose one (or a few related) problems which led to the invention of calculus (or one of the fundamental tools in calculus). You have freedom in this prompt to choose; some examples: in physics—the velocity problem, and the total displacement problem; in geometry—the tangent problem, and the area/volume problem; in economics and other applications—the net change problem. These are just a few possibilities. Whichever motivational problem you choose, you'll want to: (i) discuss the origins of the problem, including why it was a fundamental *problem* at the time; (ii) give a clear and detailed description of the problem itself—why was it difficult/novel/important for those working on it at the time? (iii) and develop how the problem was eventually solved in the context of calculus. As this is a broader prompt, there is more freedom in content and exposition than the above prompts. However, there is also more pressure to keep the writing and research focused, and make sure the paper is compelling.

  Active Link 1, Active Link 2

- *Alternative Calculuses: Multiplicative or Non-Newtonian Calculus*—This is the most difficult prompt. In this prompt you will explore alternate "calculuses". How can there be multiple calculuses? Well, our notion of calculus, developed in this class, is based on a few key assumptions and definitions. What would happen if you changed, for instance, the definition of the derivative? I would like you to explore one (or a family) of non-Newtonian or multiplicative calculuses. Make sure, in this prompt, to (i) be precise with your definitions and assumptions, (ii) describe how your non-Newtonian calculus differs from the standard calculus, (iii) perform a sample computation or two illustrating key concepts from your calculus, (iv) describe and application of non-Newtonian calculus, or at least a mathematical or real-world situation where it might be useful.

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As with any research oriented paper, in-text citation is expected, as well as a detailed bibliography should be included. (The style of annotation is not important to me, as long as it is consistent and one of the standard citation formats.) Forums and Wiki-based websites *do not constitute suitable sources for citation*, although you may certainly use them to get started. There are resources available through the library for assistance with research and citations: Active Link.