2/9/2017

Activity 1 Solutions

MATH 105, Spring 2017

Part II: Work to Do

1. What is $\dfrac{d}{dx}[3]$? Explain in a sentence why your numerical answer makes sense (using the triad notion).

   As we can see, $\dfrac{d}{dx}[3] = 0$ using the *constant rule* of differentiation. Why? Well, there are two good ways of seeing this: (i) since the constant function $y = 3$ has a graph which is a horizontal line, its tangent line is itself at all points. We know that the slope of a horizontal line is zero, and thus the slope of the tangent line to this curve is zero at all points x. Probably a better way to think about this is (ii): by definition, the constant function $y = 3$ is constant, and thus does not change. If it has not change at any point, then its instantaneous rate of change should be zero everywhere.

2. Let $f(x)$ be a function with derivative $f'(x)$. In five (or so) complete sentences, contrast the quantities: $f'(x)$, $f'(1)$, $f'(2)$, $f(1)$, and $[f(1)]'$.

   The quantity $f'(x)$ is the *derivative of* $f(x)$ as a function. Indeed, $f'(x)$ is a function, and for any x-value we want, say $x = c$, we can plug in to $f'(x)$ and what comes out is $f'(c)$ (a number), which represents the i.r.o.c. of f at $x = c$, or the slope of the tangent line to $f(x)$ at $x = c$.

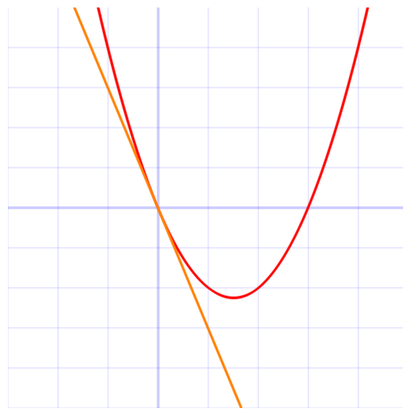
   The quantities $f'(1)$ and $f'(2)$, then, are simply slopes of two different tangent lines. One tangent line is drawn through the point on the curve $(1, f(1))$ and the other is tangent at the point $(2, f(2))$.

   The number $f(1)$ is simply the y-value associated with $x = 1$ on the graph of $f(x)$.

   Since $f(1)$ is a number (after evaluation), then using the constant rule for differentiation, we see that
   $$\frac{d}{dx}[f(1)] = 0.$$
   The point here is to note that *differentiating, then plugging in* yields a slope, while *plugging in, then differentiating* is quite silly and always produces zero.

3. According to this graph of $f(x)$ (the darker curve), what is your guess for $f'(0)$? Explain.

   Note that the line tangent to the curve at $x = 0$ goes through the points $(-1, 3)$ and $(0, 0)$. Thus it has slope $m = -3$. Since $m_{\tan}\big|_{x=0} = -3$, we know that $f'(0) = -3$.



4. Use the limit definition of the derivative (not the power rule) to find $f'(x)$ when $f(x) = x^2 - x$.

$$\begin{aligned}
f'(x) &= \lim_{h\to 0}\left[\frac{f(x+h)-f(x)}{h}\right] \\
&= \lim_{h\to 0}\left[\frac{\big((x+h)^2-(x+h)\big)-(x^2-x)}{h}\right] \\
&= \lim_{h\to 0}\left[\frac{x^2+2xh+h^2-x-h-x^2+x}{h}\right] \\
&= \lim_{h\to 0}\left[\frac{2xh+h^2-h}{h}\right] \\
&= \lim_{h\to 0}\left[\frac{h(2x+h-1}{h}\right] \\
&= \lim_{h\to 0}\left[\frac{(2x+h-1}{1}\right] \\
&= 2x-1
\end{aligned}$$

Note that if you use the power rule to compe $\dfrac{d}{dx}(x^2 - x)$, you arrive at the same conclusion.

5. Use the power rule to compute the derivatives of the following functions:

   (a) $f(x) = x = x^1$, so $f'(x) = 1x^0 = 1$.

   (b) $g(x) = x^3$, so $g'(x) = 3x^2$.

   (c) $h(x) = x^{-2.1}$, so $h'(x) = (-2.1)x^{-3.1}$.

   (d) $j(x) = \sqrt{x} = x^{1/2}$, so $j'(x) = (.5)x^{-.5}$.

   (e) $k(x) = \frac{1}{x} = x^{-1}$, so $k'(x) = (-1)x^{-2}$.

6. Look back in your notes at our old calculations of $\dfrac{d}{dx}\left[\sqrt{x}\right]$ and $\dfrac{d}{dx}\left[\dfrac{1}{x}\right]$. Write a sentence explaining whether or not you feel the power rule was easier to use.

   Using the limit definition of the derivative is cool, because it shows us that the derivative is simultaneously the instantaneous rate of change and the slope of the tangent line...however, it is cumbersome to use and often algebraically challenging. The power rule is awesome because it is easy to use, so long as we can put the function in the appropriate form.

7. Use the basic rules of differentiation from Part I to compute:

   (a) $\frac{d}{dx}\left[x + x^{-2.1}\right] = \frac{d}{dx}[x] + \frac{d}{dx}[x^{-2.1}] = 1 - 2.1x^{-3.1}.$

   (b) $\frac{d}{dx}\left[-2x^3\right] = -2\frac{d}{dx}[x^3] = (-2)(3)x^2 = -6x^2$

   (c) $\frac{d}{dx}\left[7\sqrt{x} - \frac{1}{2}x^{-1}\right] = 7\frac{d}{dx}[x^{1/2}] - \frac{1}{2}\frac{d}{dx}[x^{-1}] = 3.5x^{-1/2} + .5x^{-2}.$

8. Find the slope of the tangent line to the curve $f(x) = \frac{1}{x}$ at the value $x = 4$.

   This is simply a rephrasing of "what is $f'(4)$?" Thus $f'(4) = -x^{-2}\big|_{x=4} = -4^{-2} = \frac{-1}{16}.$

9. Find an equation of the tangent line to the curve $y = x^3$ at the value $x = -1$.

   For a tangent line, we need both a point and slope. The point should be $(-1, f(-1) = (-1, (-1)^3) = (-1, -1)$. The slope of the tangent line at $x = -1$ should be $f'(-1)$, which is

   $$f'(-1) = 3x^2\big|_{x=-1} = 3(-1)^2 = 3.$$

   Putting it all together, the equation for the tangent line should be:

   $$y-c = m_{\tan}\Big|_{x=c}(x-c) \iff y-(-1) = 3(x-(-1)) \iff y+1 = 3(x+1) \iff y = 3x+2.$$